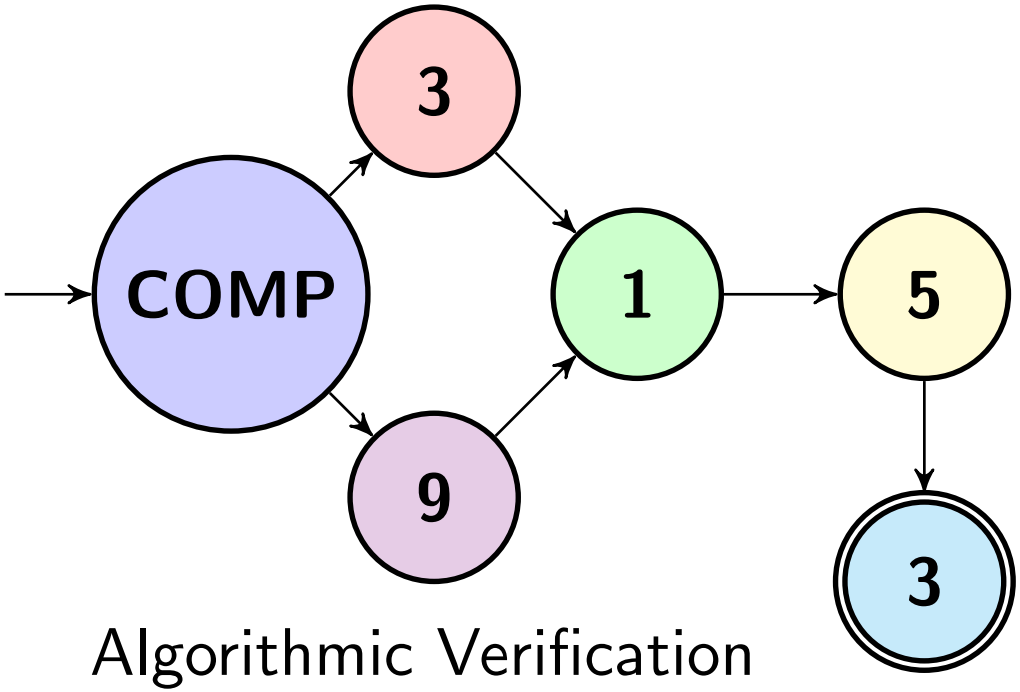

Algorithmic Verification

Simulation and Bisimulation

Dr. Liam O'Connor
CSE, UNSW (for now)
Term 1 2020

Model Equivalence

Let A and B be Kripke structures.

Question

When does $A \models \varphi \Leftrightarrow B \models \varphi$ for all LTL formulae φ?

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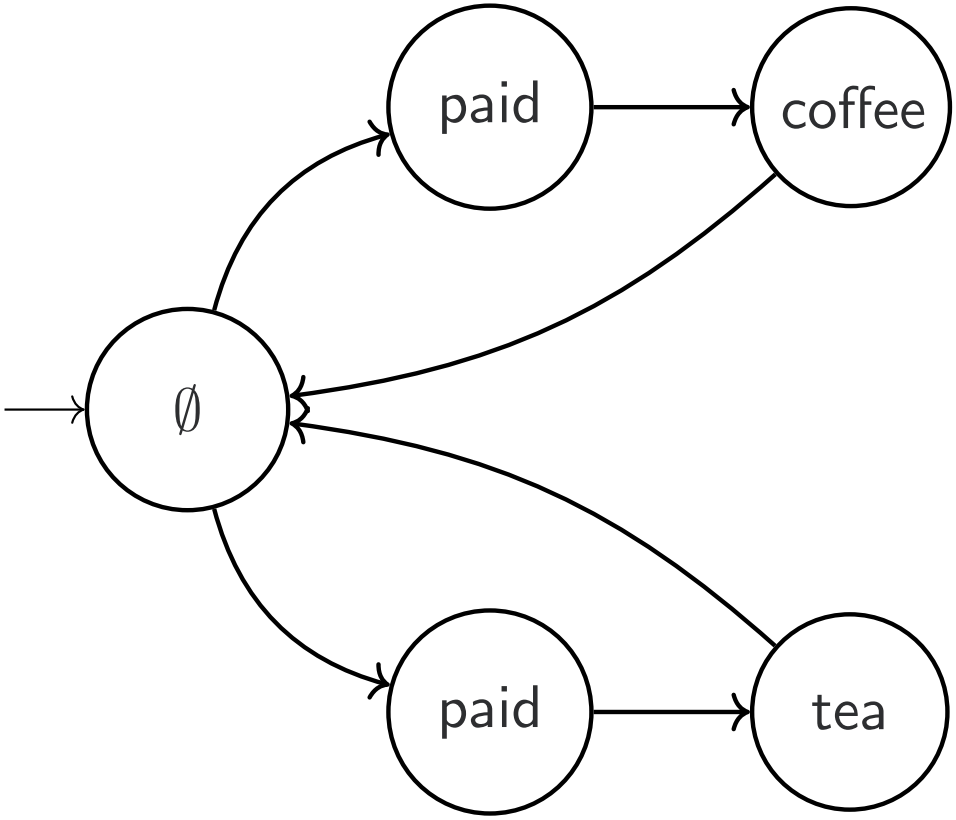
This is called *infinite completed trace equivalence*.

Limitations of Traces



Traces cannot distinguish these two models!

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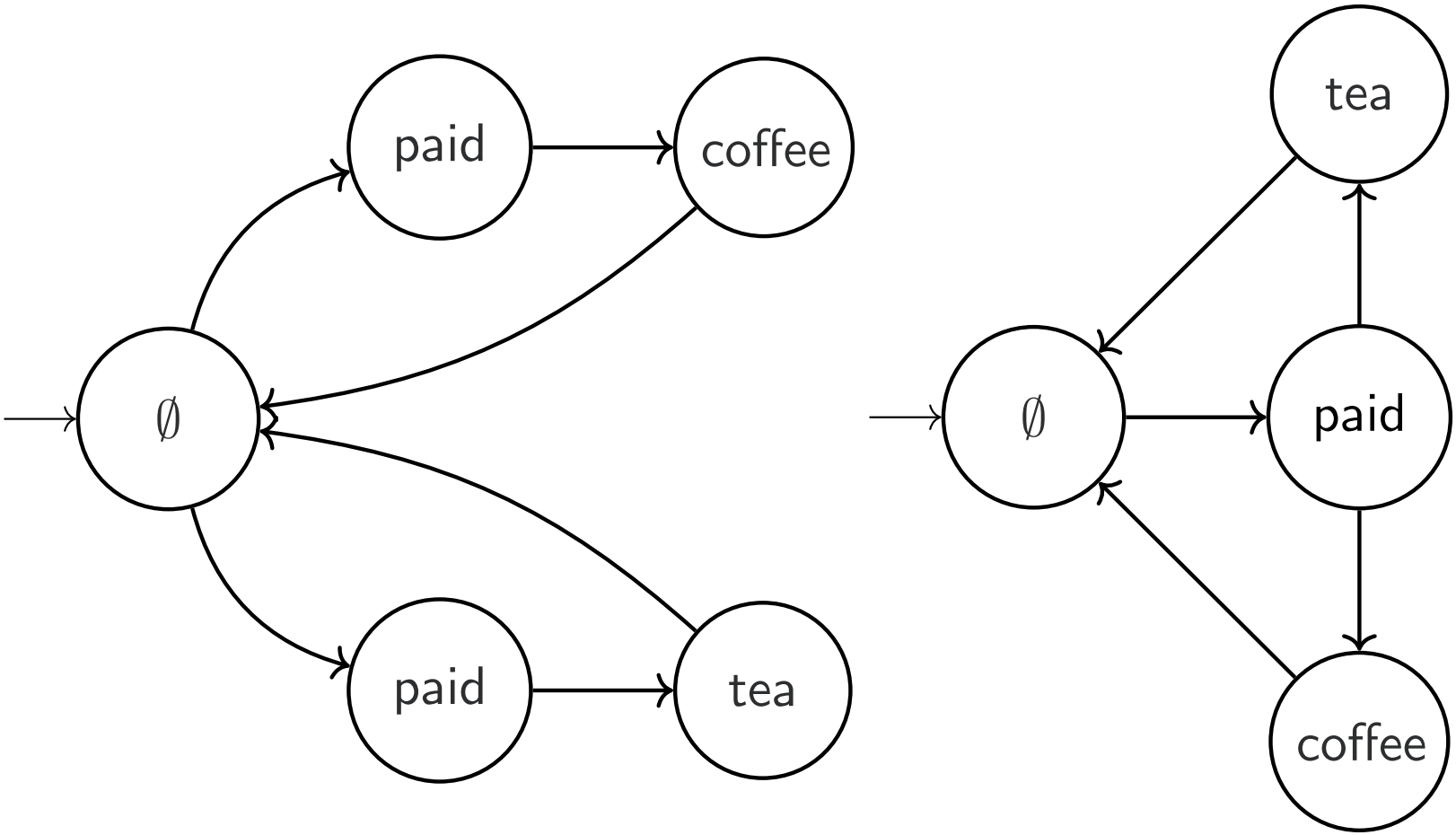

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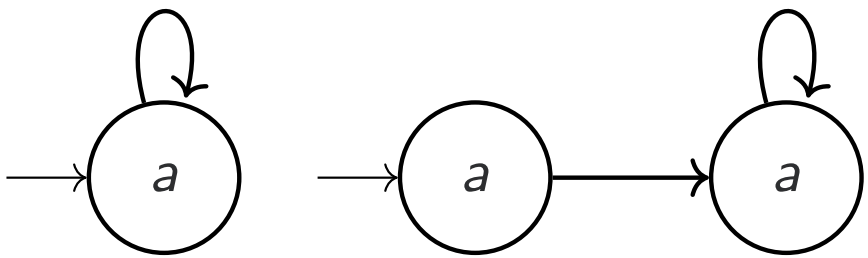

Nope!

Tree Equivalence?

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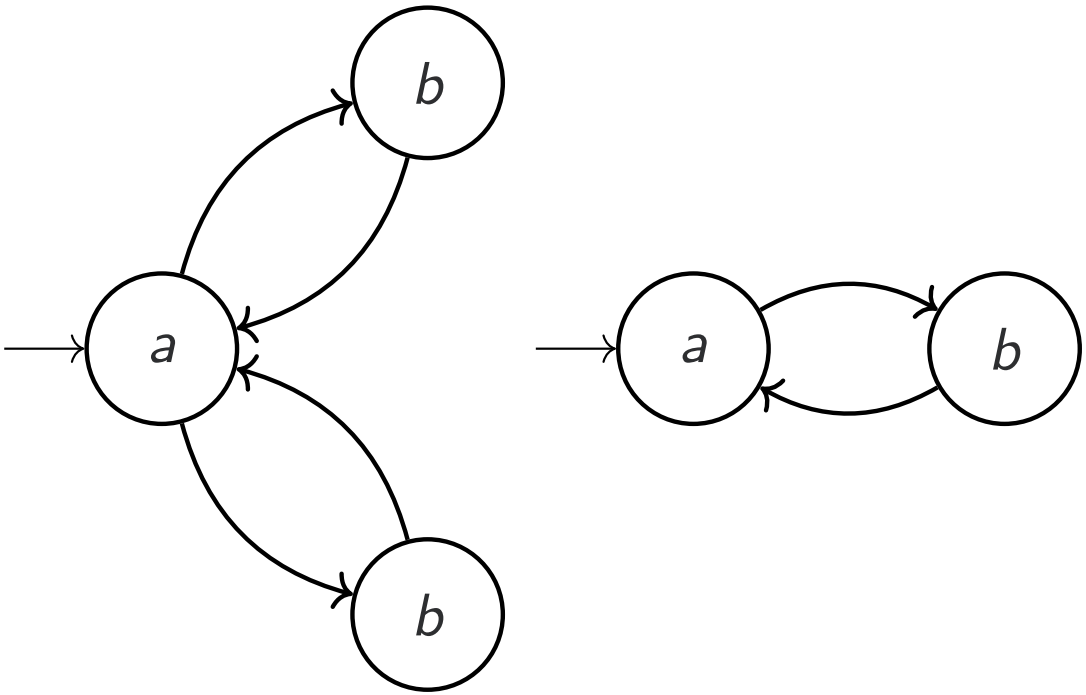

Also no!

Bisimulations

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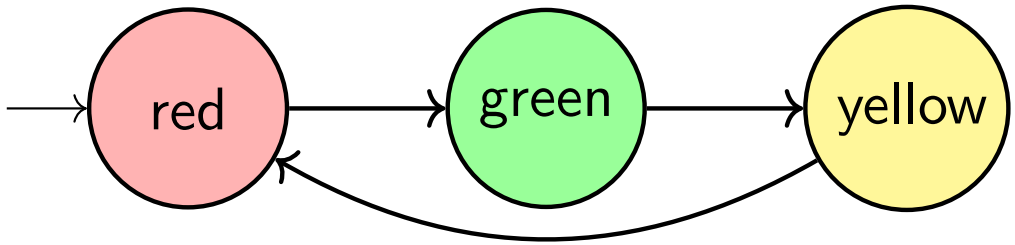
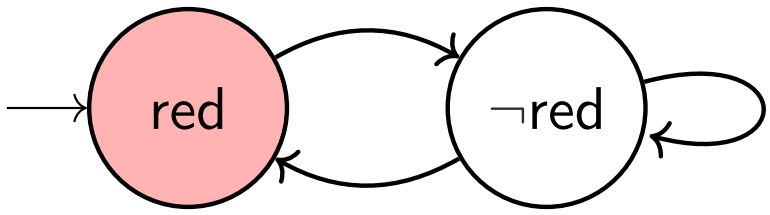
Result

For two finitely-branching automata A and B, $A \models \varphi \Leftrightarrow B \models \varphi$ for all CTL formulae φ iff they are *bisimilar*.

Simulation





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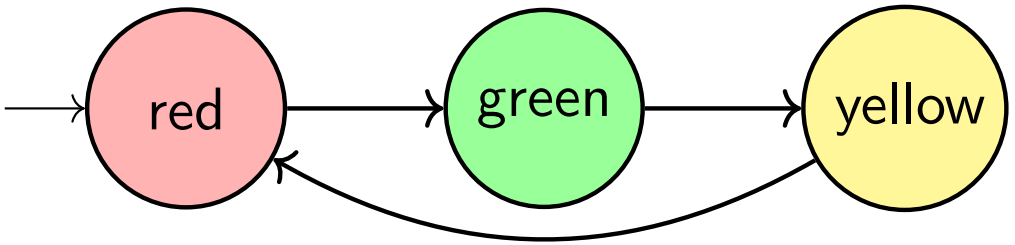

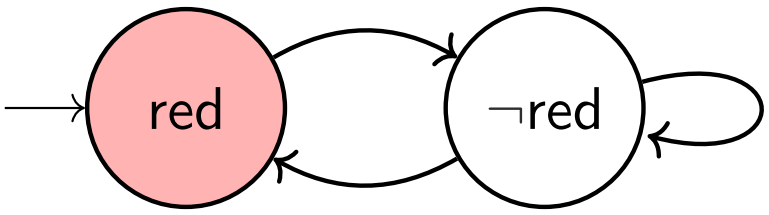

Are these bisimilar?

No, but one simulates the other.

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Abstraction and Traces

If $A \sqsubseteq C$, then every trace of C restricted to L_A is a trace of A.

$$\sigma_1\sigma_2\sigma_3 \cdots \in \text{Traces}(C)$$

$$\Rightarrow$$

$$(\sigma_1 \cap L_A)(\sigma_2 \cap L_A)(\sigma_3 \cap L_A) \cdots \in \text{Traces}(A)$$

Essential Property of Simulations

Let $\mathcal{A}$ be a simulation relation, showing that $X \sqsubseteq Y$.
Then for every run $\rho_1\rho_2\rho_3\cdots \in Y$ is a run of X by applying the simulation relation as an *abstraction mapping*:

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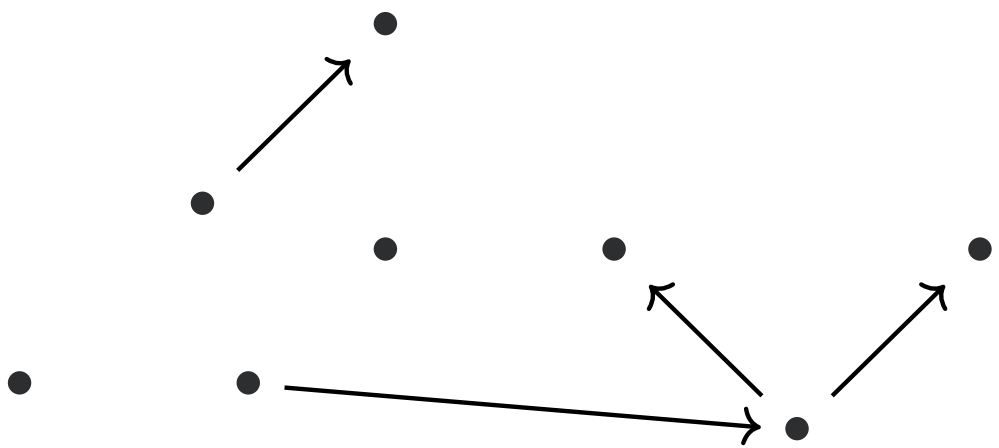

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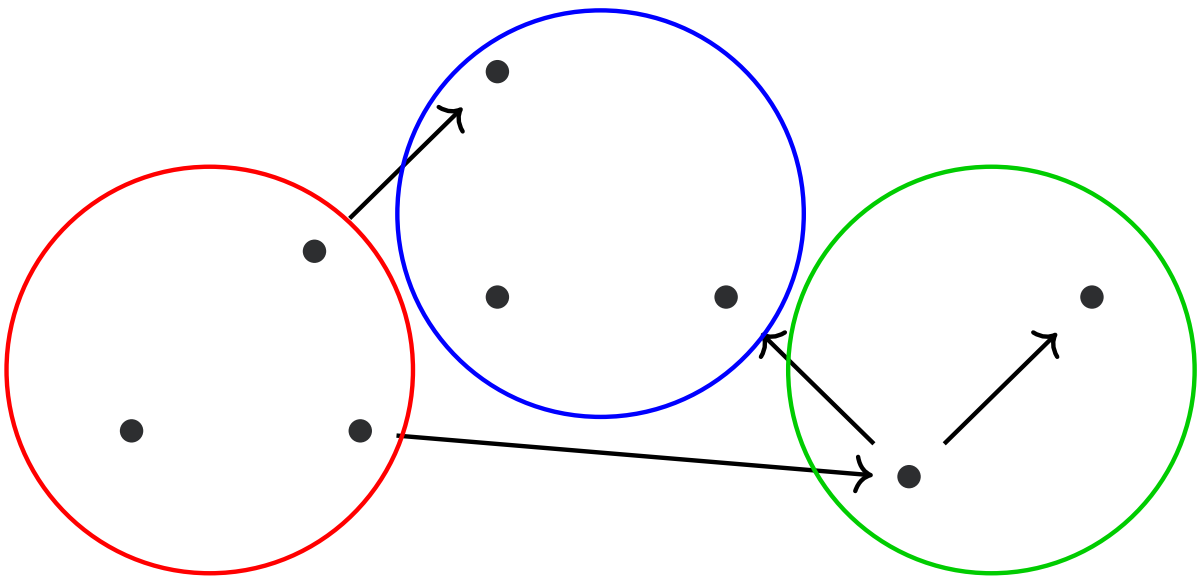

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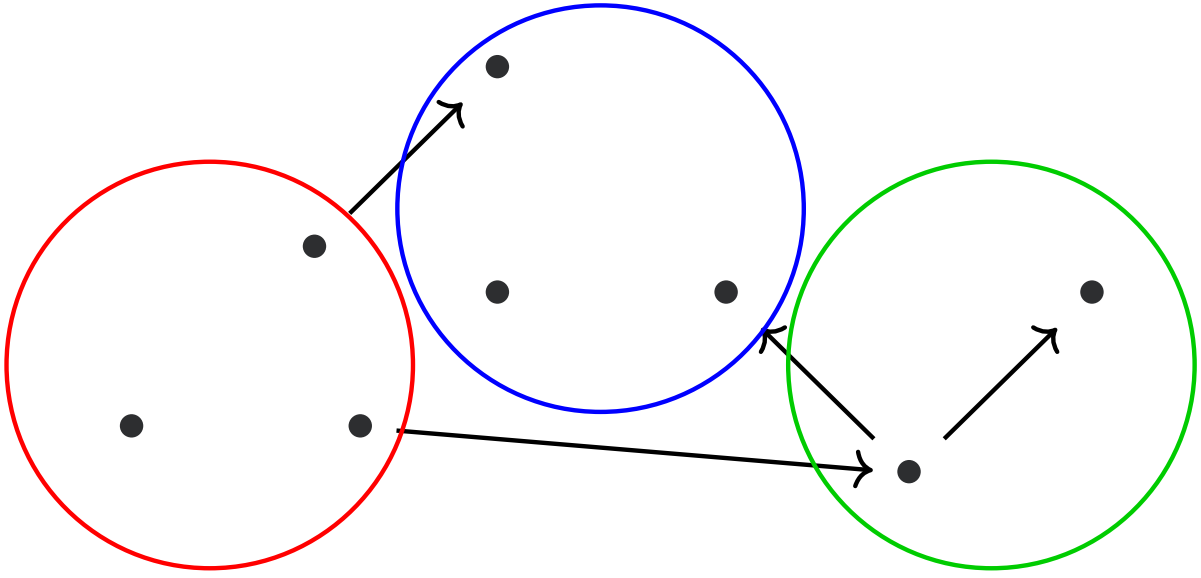
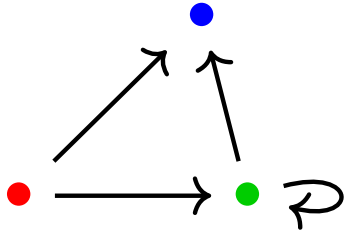
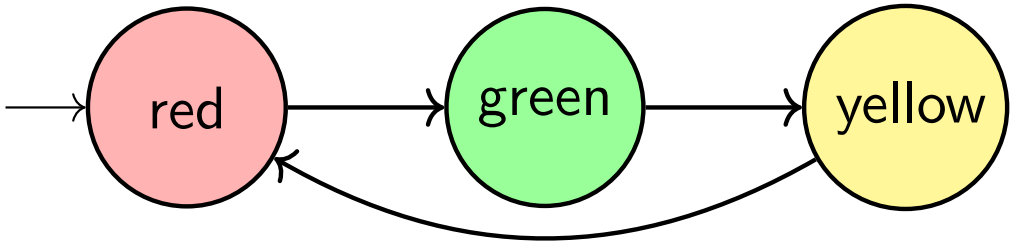
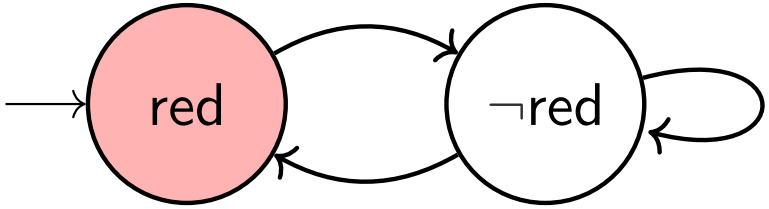
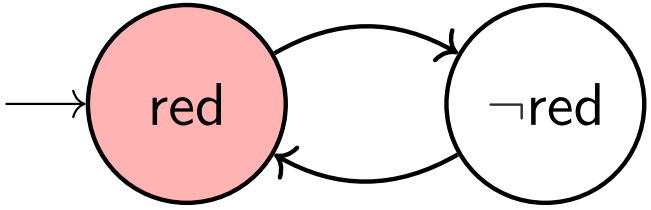


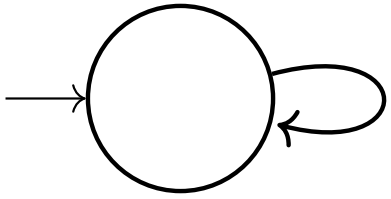


Comparing Automata









What are the simulations between these?

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We want abstraction to shrink the state space for model checking. To do this, we need a guarantee that any property we prove about an abstraction applies just as well to the concrete model.

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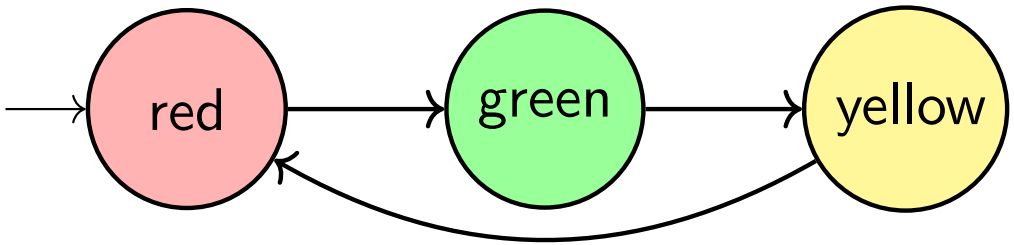

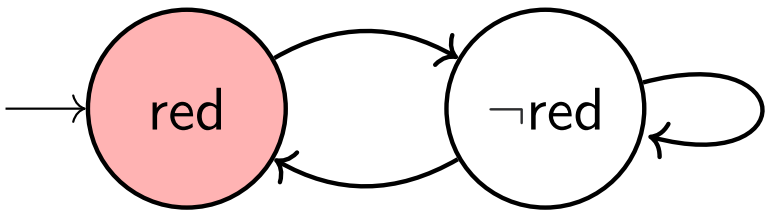

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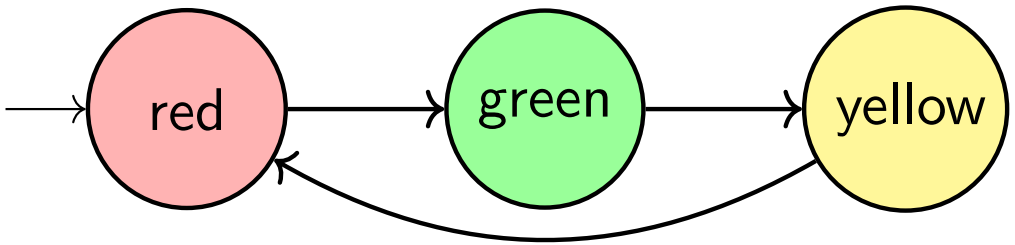

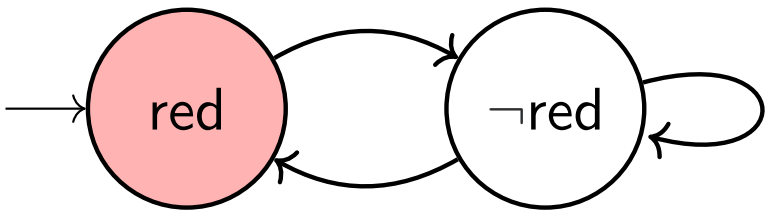

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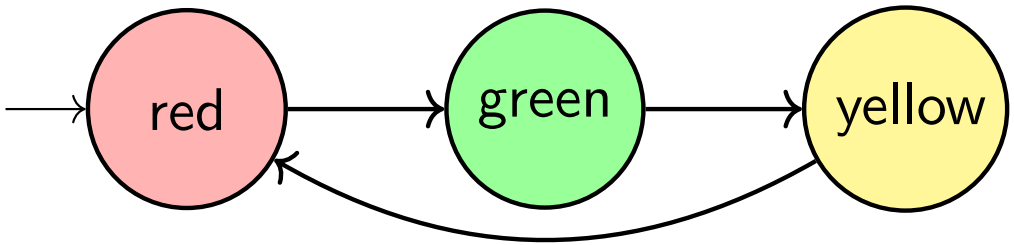

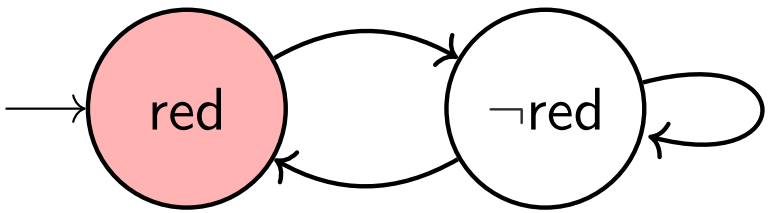

AG AF ¬red? Works!

¬**AG AF** red? Doesn't work!

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Example

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- **AG AF**p
- **EF**p — Nope!

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$$
\begin{aligned}
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\end{aligned}
$$

Release Operator

The temporal operator $\varphi\ \mathbf{R}\ \psi$ says that ψ will not become false unless φ happens first.

$$\sigma \models \varphi\ \mathbf{R}\ \psi \quad \Leftrightarrow \quad \forall n \geq 0.\ (\forall 0 \leq k < n.\ \sigma|_k \not\models \varphi) \Rightarrow \sigma|_n \models \psi$$

A and **E** variants in CTL follow the usual pattern.

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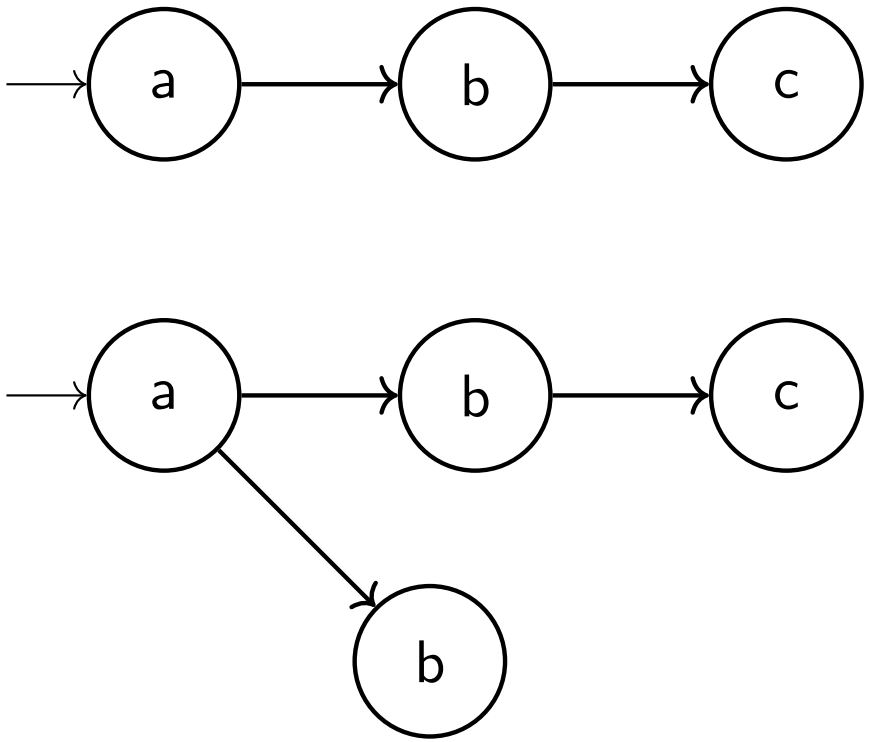

Nope! This is another equivalence called *simulation equivalence*. Because of the abstraction result, ACTL is the logic that characterises simulation equivalence.

The Linear-time Branching-time Spectrum

Coarseness of Equivalences

- Graph isomorphism is finer (distinguishes more models) in than bisimilarity.

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The Linear-time Branching-time Spectrum

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Rob van Glabbeek categorised all of these equivalences and more into the *linear-time branching-time spectrum*, which is a major focus of his course at this university, COMP6752.

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